

Syzygies: Geometry, Combinatorics and Complexity I, II, and III

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Abstract

I will discuss several classic and recent (some conjectural) geometric and combinatorial bounds for the complexity of the equations and syzygies of a projective variety, as expressed by the Castelnuovo–Mumford regularity of their defining ideal. From an algebraic point of view the simplest projective subschemes are those whose homogeneous ideals are generated by quadrics with a long strand of linear syzygies. I will describe geometric and combinatorial consequences from the presence of such a long strand, and discuss recent results and work in progress relating the geometry and the combinatorics of homogeneous ideals and projective schemes of small regularity, with a focus on the open subset of the Hilbert scheme parameterizing 2-regular schemes. These results extend the more than 100 years old Del Pezzo–Bertini classification of varieties of minimal degree, and the modern characterization of these as the varieties of Mumford–Castelnuovo regularity 2.